

Omnidirectional Gravitational Wave Detector with a Laser-Interferometric Gravitational Compass

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Abstract

Based on the Szekeres-Pirani gravitational compass we suggest the addition of a fourth, non-coplanar mass/mirror to the presently existing laser based gravitational wave observatories, enabling them to operate omnidirectionally, to filter out ambiguous interpretations and to point out the direction of the gravitational wave source.

1 Introduction

The recent detection of gravitational waves in the event GW150914, clearly shows the expected waveforms produced by a coalescent black hole binary system [1]. The sophisticated upgrade implemented in those detectors proved to be adequate to produce the very small shifts of the interferometer fringes, of the order of two hundredth of the proton diameter.

The theory behind the gravitational wave observatories, is based on the well known geodesic deviation equation [2]. Although this equation by itself is a purely geometrical statement in Riemannian geometry, its application to the space-times of General Relativity provides an intuitive view on how the gravitational field affects the displacement of test particles in terms of the variations of the curvature of the space-time in their immediate neighborhood. To see how this works, consider initially two massive test particles at positions A and B, subjected to the Earth's gravitational field. The particle at A sends a signal with velocity P to the particle at B along the shortest possible distance, meaning that the signal travels along a geodesic with equation $\nabla_P P = 0$. Immediately after a signal is sent, both particles are allowed to fall freely (that is, under the exclusive influence of the gravitational field), along time-like geodesics with tangent vectors T and T' respectively, with T' parallel to T , also satisfying geodesic equations $\nabla_T T = 0$ and $\nabla_{T'} T' = 0$. After a while, the particles reach positions A' and B', where B' is reached by the signal emitted from the particle at A'. Since the signal and fall speeds

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are different, the first needs to vary along the fall, at a rate such that at the end we obtain a closed geodesic parallelogram satisfying the closing condition $\nabla_T P = \nabla_P T$. In operator form, the Riemann tensor R evaluated in such parallelogram gives

$$R(T, P)T = [\nabla_T, \nabla_P]T = \nabla_T(\nabla_P T) - \nabla_P(\nabla_T T) = \nabla_T(\nabla_T P). \quad (1)$$

Denoting by $a = \nabla_T P$ the acceleration of the signal along the free fall, this gives the geodesic deviation equation (GDE)

$$\nabla_T a = R(T, P)T. \quad (2)$$

A more familiar form of this equation follows from its application to a reference frame defined by base vectors $\{e_\mu\}$ with $\mu = 0, 1, 2, 3$, in which the components of the Riemann tensor are given by $R(e_\mu, e_\nu)e_\rho = R_{\mu\nu\rho\sigma}e^\sigma$. In particular, taking $T = e_0$ as the time direction, and $P = e_i$, where e_i , is any one of the space-like base vectors, (2) becomes

$$\frac{da^i}{cdt} = R_{0i0i}, \quad \text{fixed } i, \quad (3)$$

where c is the speed of light. Thus, the falling pair of particles describe a 2-dimensional world-sheet in space-time with signature $(-, +)$, whose curvature tensor has a single component R_{0i0i} , producing the variation of the signal acceleration in the left hand side.

For gravitational wave detection we use the same reasoning, where again the two particles are initially at rest, either floating in the void, or properly suspended to compensate for the Earth's gravitational pull. When they are hit by a gravitational wave train, instead of experiencing a "free fall", they will "surf the gravitational waves" along geodesics of the locally distorted space-time geometry. As the geodesic parallelogram closes, we obtain the same equation (2), defined in the world-sheet generated by the geodesics with tangent vectors T and P . Again, using a reference frame with $T = e_0$ and $P = e_i$, for any space-like vector e_i , we obtain the same equation (3) describing the geodesic deviation in the the 2-dimensional world-sheet generated by $\{e_0, e_i\}$.

As it happens, any 2-dimensional Riemannian (or pseudo-Riemannian) manifold is conformally flat. This means that it is always possible to find a function φ such that its metric can be written as $g_{ij} = e^{2\varphi}\eta_{ij}$ and $g^{ij} = e^{-2\varphi}\eta^{ij}$, where η_{ij} denotes the 2-dimensional metric for $i, j = 1, 2$. Calculating the Ricci tensor for g_{ij} we obtain $R_{ij} = (\varphi_{,11} - \varphi_{,22})\eta_{ij}$ and the Ricci scalar gives $R = 2e^{-2\varphi}(\varphi_{,11} - \varphi_{,22})$. It follows that $R_{ij} \equiv \frac{1}{2}Rg_{ij}$, so that Einstein's equations vanish identically in any 2-dimensional space-time. Consequently, the interpretations of a gravitational wave event in detectors constructed with only two test masses may require considerations on 2-dimensional gravitational theories which are not derived from Einstein's equations, like in the Jackiw-Teitelboim, in the Liouville gravity and in string-like theories [3].

The presently existing interferometric gravitational wave detectors based on three, non-coplanar, test masses (e. g. like GEO600, VIRGO and LIGO) represent an improvement of the sensibility of the 2-mass systems obtained by the addition of one extra test mass, or equivalently, by increasing the dimension of the previous world-sheet to a 3-dimensional world-volume with signature $(-, +, +)$ within the four-dimensional space-time, in which the geodesic deviation takes place. Actually, the LIGO equipments use 4 masses, with two at the extremes of each arm L_1 and L_2 and two extra masses located near the intersection of those arms, all positioned in the same plane, so that for point of view of the geodesic deviation equation effectively takes place in a 3-dimensional space-time.

After being hit by a gravitational wave front, the deformation of the space-time geometry is given by the components of the Riemann tensor of the 3-dimensional space-time volume generated by the motion of the 3 masses. The geodesic deviation equation describe the variation of the communication signals between the three masses as compared with the initial positions. Two geodesic signals appear, with accelerations denoted by a_{ij} , with $i, j = 1, 2$. This may also be expressed in terms of the frequency shifts, $e^{i\omega_{ij}}$, so that (3) reads"

$$\frac{1}{\omega_{ij}} \frac{d\omega^{ij}}{cdt} = R_{0i0j}, \quad i = 1, 2. \quad (4)$$

Although there is an improvement with respect to the 2-mass detectors, the restriction of General Relativity to an effective 3-dimensional space-time subset implies that the curvature changes takes occurs with a smaller number of degrees of freedom as compared to the complete four dimensional General Relativity. This is a consequence of the fact that the Riemann tensor in 3-dimensions is determined by the Ricci tensor [4]. Therefore, when Einstein's equations are applied, the Riemann curvature becomes essentially determined by matter, so that in the case of empty space, the Riemann tensor vanishes. However, it is possible to restore a curvature term in empty space, by the use of a non-trivial (Chern-Simmons) topological construction, in association with topological BTZ black-holes [5, 6]. Since the LIGO detector falls in the category of 3-mass system, the event GW140915 may correspond to a genuine gravitational wave from four-dimensional Einstein's relativity, but it does not exclude the possibility that a 2+1 dimensional topological gravitational theory may have occurred. However, this is something still to be checked.

2 The Gravitational Compass

The observables of Einstein's gravitational field are given by the eigenvalues of the Riemann tensor

$$R_{\mu\nu\rho\sigma}X^{\mu\nu} = \lambda X_{\rho\sigma}, \quad (5)$$

where $X^{\mu\nu} = X^{[\mu\nu]}$ is a two-form (or a bi-vector or a skew-vector) eigenvector [7, 8]. In a four-dimensional space-time, there are at most six independent eigenvectors, solutions of (5). These eigenvectors can be written in the Newmann-Penrose null frame to determine the six Petrov types O, I, II, III, N and D of the the gravitational field. Type O corresponds to zero eigenvalues, so that we have only five effective Petrov types corresponding to the five non-trivial observable degrees of freedom of the gravitational field.

On the other hand, the eigenvectors of the curvature tensor generate a six dimensional space, which can be set in a 1:1 correspondence with six linearly independent 3×3 matrices. Using such matrices, it is possible to determine the principal curvature directions of the gravitational field [9]. This is the basis of Peter Szekeres' proposition of a device called *the gravitational compass*, consisting of four non-coplanar masses, labeled O, A, B, C, connected by six dynamometers whose forces measure the eigenvalues of the gravitational field (fig. 1 below). The compass could in principle be rotated around the mass at O, so that the forces on the diagonal dynamometers AB, AC, BC vanish, and the radial dynamometers OA, OB, OC indicate the principal directions of the curvature tensor [10]. The gravitational compass offers a practical measurement of the true relativistic gravitational field, by simply placing such device in a region where the presence of a gravitational field is measured through the forces given by the six dynamometers.

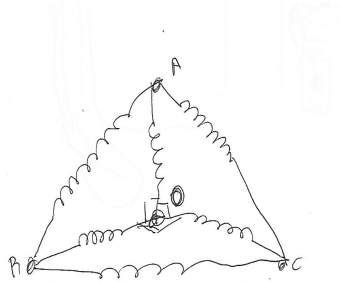


Figure 1: Szekeres' Gravitational Compass

The resonant-mass spherical gravitational wave detector constructed with a solid sphere made of an elastic material can be thought as a practical realization of Szekeres' compass, where the dynamometers are replaced by the tensions resulting from the principal modes of oscillation of the spherical surface [11, 12].

A simpler device of the same nature of the Szekeres compass, was proposed by Pirani, in which the dynamometers are replaced by a telemetric measure of the relative displacements of the four points of the Szekeres gravitational compass [2]. Since four points in the 3-dimensional space determine a spherical surface, the Szekeres-Pirani gravitational compass could provide a practical measure of the gravitational field by observing the principal modes of oscillation of the four masses, or equivalently of the sphere, using laser interferometers. Such modes are illustrated below by the graphical display of the p-modes of a neutron star oscillation [13]. The gravitational compass owes its name to its ability to find the direction of a given source of gravitation, by

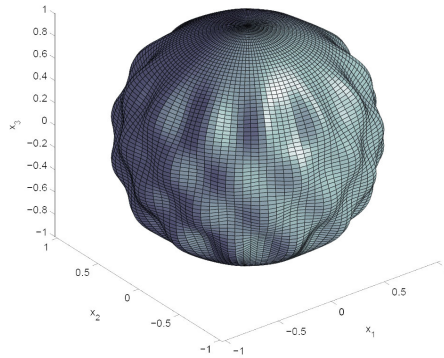


Figure 2: Oscillations in a spherical surface.

determining the principal directions of the curvature tensor. Using the symmetry group of the above mentioned matrices, we can find that when the forces along the diagonal dynamometers AB, AC and BC vanish, the forces along OA, OB and OC correspond to the principal directions of the curvature tensor, making it a natural device to measure gravitational radiation, pointing out to the direction of the source. Thus, in principle it is possible to construct an omnidirectional interferometric gravitational detector by extending some of presently existing laser-interferometric gravitational wave detectors through the addition of a fourth mass outside the plane of the three existing ones [14].

As an example, the GEO600 detector in Hanover, Germany, could be upgraded to a gravitational compass by adding a 600m tower in the same detector site (The construction of such towers made of metal mesh supported by an array of stays has proven to be feasible by the operating 650m radio antenna in Poland.). Of course, some additional engineering challenges would be in the way, such as the inclusion of a well insulated tube for the laser to compensate for the temperature gradient, the suspension of the mirror at such altitudes, and the swing of the towers caused by strong winds. Alternatively, instead of a tower a deep 600m well could be dug, but this would probably need a new construction site with additional costs and engineering adaptations.

In principle the Szekeres-Pirani Gravitational Compass may also be applied to the planned LISA space detectors of gravitational waves, which would have the same limitations of the 3-mass plane geometry of the present laser detectors. In order to make the detector omnidirectional a fourth satellite would be required, again defining a spherical-like detector in space. Admittedly, the stability of a four body system in space may represent an additional problem as the satellites may depend of a propulsion system for the orbit stability at least for long term use.

Summarizing, the observables of the gravitational field are given by the five non-trivial eigenvalues of the

curvature tensor of space-time. Therefore an efficient gravitational wave detector truly in conformity with Einstein's gravitational theory must be able to detect the corresponding five degrees of freedom of Einstein's gravitational field. With basis on the Szekeres-Pirani Gravitational compass we suggest that the existing GEO600 laser-interferometric gravitational wave detector could be converted to a four test-mass gravitational wave detector, thus becoming the prototype for omnidirectional interferometric detectors. As well, we suggest that planned interferometric detectors, could be designed according to the gravitational compass concept from the beginning in order to be a fully omnidirectional gravitational wave observatory by itself".

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